Electron Plasma Waves on Cylindrical Structures and the Concept of an Equivalent Dielectric

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Z. Naturforsch. **50a**, 897–901 (1995); received May 22, 1995

The dispersion relation $\Delta_0(\omega, \beta)=0$ of the azymuthally symmetric electron surface wave propagating on a plasma column placed in a glass tube of thickness δ and permittivity ε_g has a fairly complicated form, with several parameters involved in many clusters of Bessel functions of various orders, kinds and arguments. Much simpler is the dispersion relation $\Delta_0^{\mathrm{h}}(\omega, \beta)=0$ for the surface wave on a plasma cylinder in a homogeneous dielectric, but that situation is often inapplicable as a model in real experiments. However, one can reproduce the Δ_0 dispersion curves starting from the Δ_0^{h} ones, conveniently choosing an equivalent glass permittivity $\varepsilon(\delta, \varepsilon_g, \beta)$ instead of the constant ε_g. We have proved that this procedure gives acceptable approximate results. The computations valid for an azimuthally symmetric surface wave are given and the necessary comparisons done.

1. Introduction

An immense experimental work on the propagation of surface waves along a plasma/dielectric interface has been done during the past two or three decades. The results and published papers are partly summarized and reviewed [1, 2]. Many articles among them deal with the surface waves in gas-discharge plasmas [3]. As a rule, in such cases the plasma is created by ionizing a gas or a vapour within a dielectric tube (a glass of permittivity ε_g) of known wall thickness δ. A similar situation exists in experiments with surfatron plasma sources [4, 5]. In all these experiments, the involved geometries are cylindrical with the typical *plasma column/glass tube/air* structure. Theoretical papers often follow that fact, in both linear [3] and nonlinear treatments [6].

The dispersion relation $\Delta_0(\omega, \beta)=0$ of an azimuthally symmetric surface wave on a lossless cold plasma column surrounded by a glass cylinder of finite thickness and circular cross section implies a function of a fairly complex form (see the Appendix):

$$\begin{aligned}&m_{13} m_{24} m_{31} m_{42} - m_{11} m_{24} m_{33} m_{42}\\&- m_{12} m_{24} m_{31} m_{43} + m_{11} m_{24} m_{32} m_{43}\\&- m_{13} m_{22} m_{31} m_{44} + m_{12} m_{23} m_{31} m_{44}\\&- m_{11} m_{23} m_{32} m_{44} + m_{11} m_{22} m_{33} m_{44} = 0. \quad (1)\end{aligned}$$

Even in the simplifying case of the quasistatic approximation, when the expression achieves the explicit structure of the type $\omega=f(\beta)$, the involved mathematics is still very complicated – the relevant parameters (permittivities, geometrical factors) exist in clusters of modified Bessel functions I_0 and K_0 and related derivatives I_0', K_0', partly in their arguments. Denoting $x=\beta a$, $z=xd$, $y=\omega/\omega_p$ and $S(z)=K_0'(z)\, I_0(z) - \varepsilon_g K_0(z)\, I_0'(z)$, we obtain from (1) the quasistatic formula (cf. [7])

$$1-\frac{1}{y^2}-\varepsilon_g \frac{I_0}{I_0'(x)} \cdot \frac{(1-\varepsilon_g) I_0'(x) K_0(z) K_0'(z) - K_0'(x) S(z)}{(1-\varepsilon_g) I_0(x) K_0(z) K_0'(z) - K_0'(x) S(z)}=0. \quad (2)$$

Here, primes denote differentiations with respect to x, a and b are the radii of the tube and $d=b/a$.

On the other hand, much simpler is the dispersion relation $\Delta_0^{\mathrm{h}}(\omega, \beta)=0$ of the surface wave on a plasma column placed in a homogeneous, endless dielectric, i.e. in the limit $d=\delta/a+1 \to \infty$. In the same approximation and for the same wave mode, we have (cf. [7])

$$y=\frac{1}{\sqrt{1-M(x)\, \varepsilon_g}}, \quad (3)$$

where $M(x)=I_0(x)\, K_0'(x)/[I_0'(x)\, K_0(x)]$. Using the above relations, we can plot the dispersion curves. Let us show and comment on some of them in order to explain our idea, which shell be elaborated in the next Section, in the most convenient way. The curve a), Fig. 1, is valid in the limit $\varepsilon_g \to 1$ or, which is the same, when $d \to 1$ (the tube is of negligible thickness). The curve b) represent the case of a very thick tube wall,

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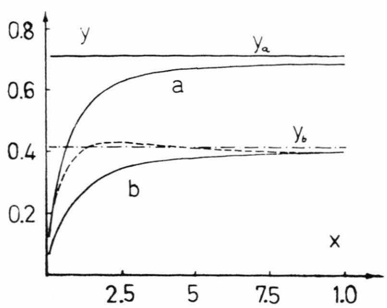

Fig. 1. The dispersion curves for the azimuthally symmetric wave; a) valid in the limit $\varepsilon_g \to 1$; b) valid in the limit $d \to \infty$ and $\varepsilon_g = 4.8$; the dashed line: $d = 1.2$ and $\varepsilon_g = 4.8$; the horizontal lines are the asymptotes for $x \to \infty$.

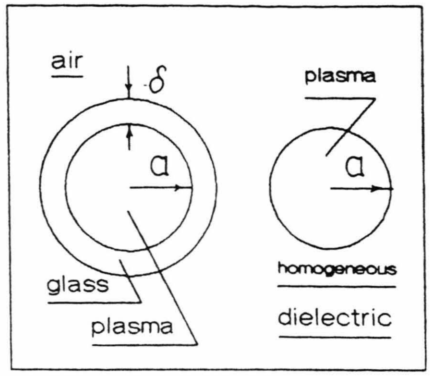

Fig. 2. The real experimental configuration (left) is to be replaced with the simpler model (right) without changing the dispersion relation.

$d \to \infty$. In the limit $x \to \infty$, the asymptotes are, respectively, y_a(air) $\simeq 0.707$ and y_b(glass) $\simeq 0.415$, both values being in accordance with the common expression $y_{asym} = 1/\sqrt{1+\varepsilon_g}$. The dashed line in the Figure 1 gives the dispersion for $d = 1.2$, the points having been computed by means of (1). We see that the dashed line can practically not be resolved from curve a) for small normalized wavenumbers x and that it tends to join wiht curve b) for large values of x. This has a simple physical explanation: 1) for small wavenumbers the surface wave wavelength is large in comparison with the wall thicknes, $\lambda/\delta \gg 1$ and the wave "feels" only the air (vacuum) permittivity; 2) for large wavenumbers, on the contrary, we have $\lambda/\delta \ll 1$ and the wave "feels" mainly the glass as the medium surrounding the plasma column. For middle values of x the wavelength is comparable with the thickness, and the dispersion curve must have a maximum and a region with negative $\mathrm{d}y/\mathrm{d}x$. In fact, the curve exhibits a broad minimum, too, since it approaches the asymptote from its lower side. In any case, in the neighborhood of the point $x = 1$ the situation is more complicated: the wave "feels" the glass permittivity as well as the air permittivity, i.e. the physical picture can be understood by means of some equivalent dielectric representing the glass/air combination. Let us see what effective permittivity must have this equivalent dielectric. As far as we know, this problem has not been analysed yet.

2. Equivalent Dielectric

In order to answer the above question, we shall analyse the substitution sketched in the Figure 2. The problem is to find the effective permittivity $\varepsilon = \varepsilon(d, \varepsilon_g, x)$ of the homogeneous dielectric which ensures the identity of the dispersion relations for the two configurations.

The physical conditions that must be applied when formulating the permittivity ε are as follows: a) $\lambda \to \infty$ must result in $\varepsilon \to 1$; b) when $\lambda \to 0$ then $\varepsilon \to \varepsilon_g$. In addition, the dependances on the thickness and the wavenumber should be in the form of the product: $\varepsilon = f(\delta\beta)$. The reason is that the crucial "measure" of the wall thickness is the wavelength of the surface wave. In other words, as the argument in the ε function appears the variable $\eta = (d-1)x$, where $d = b/a$ is the quotient of the tube radii and $x = \beta a$. Therefore one could reasonable suggest the form

$$\varepsilon = 1 + E_1(\varepsilon_g) \cdot E_2(\eta). \tag{4}$$

The function E_1 and E_2 must satisfy the requirements of the mentioned physical conditions. The list of the requirements is:

1. $\varepsilon_g \to 1 => E_1 \to 0$,
2. $\eta \to 0 => E_2 \to 0$,
3. $x \to \infty => E_2 \to 1$,
4. $x \to \infty => E_1 \to (\varepsilon_g - 1)$.

Of course, $\eta \to 0$ can be realized in the two ways: $x \to 0$ and $d \to 1$. Also, $\eta \to \infty$ can be realized in the two ways: $x \to \infty$ and $d \to \infty$.

3. Effective Permittivity

We have found that the satisfactory choice is $E_1 = \varepsilon_g - 1$ and $E_2 = \tanh(\eta)$, when the effective permittivity takes the form

$$\varepsilon(d, \varepsilon_g, x) = 1 + (\varepsilon_g - 1) \tanh[(d-1)x]. \quad (5)$$

Figure 3 shows the dispersion curve obtained a) from (1) and b) from (2) with the substitution $\varepsilon_g \to \varepsilon$ in accordance with the above model. The values of curve a) are slightly larger in the region $x \sim 2$; the differences are practically undetectable for $x < 1$ and $x \gg 1$. The discrepancies decrease with the increase of d, and if $d > 1.5$ the approximate curve is rather identical with the original one. Figure 3 holds for $\varepsilon_g = 4.8$ (pyrex; the example chosen to match the experimental conditions met by the surface wave group in Belgrade, Yugoslavia), but the conclusions are valid for other sorts of glasses, too. In fact, the equivalent dielectric with its effective permittivity defined by (4) is applicable in case of most experiments ($\varepsilon_g \sim 4$, $d \sim 1.2$, $x \sim 1$). Of course, in the case when an another dielectric spreads around the glass tube (instead of air), the generalization of (4) can easily be done.

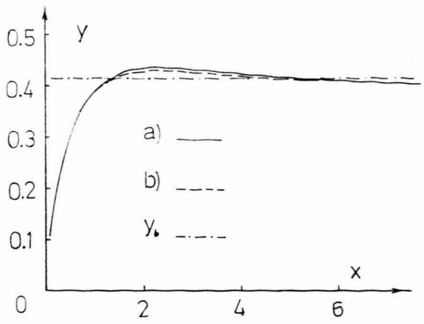

Fig. 3. The quasistatic dispersion curve (a) compared with the approximate one (b).

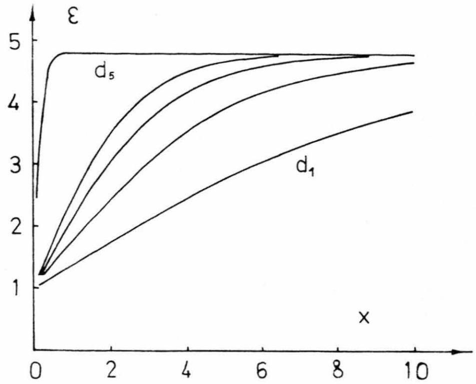

Fig. 4. The effective permittivity as a function of the normalized wavenumber; the parameter d takes the values 1.1 (the label d_1), 1.2, 1.3, 1.4, and 5 (the label d_5).

4. Approximations

Let us now see in the Fig. 4 the effective permittivity as a function of the normalized wavenumber, for various values of d. The family of curves enables the finding of the right "working point", as one sees the impact of the wall thickness on the surface wave features. The Fig. 5 shows the dependance of the effective permittivity on the wall thickness parameter for several normalized wavenumbers. Such graphs are convenient to formulate the criteria for approximations. Let us now consider what tube can be analyzed as a thick-wall, and what as a thin-wall one.

The thick-wall approximation. Among many possible approaches we will proceed as follows. First of all let us define the permittivity ε_+ by means of the relation $\varepsilon_+ = 0.9\,\varepsilon_g$. For a given value of x, the equality $\varepsilon = \varepsilon_+$ is valid if the thickness parameter takes the value d_+, which must be calculated from the expression

$$x(d_+ - 1) = \operatorname{arctanh}\left(\frac{0.9\,\varepsilon_g - 1}{\varepsilon_g - 1}\right). \quad (6)$$

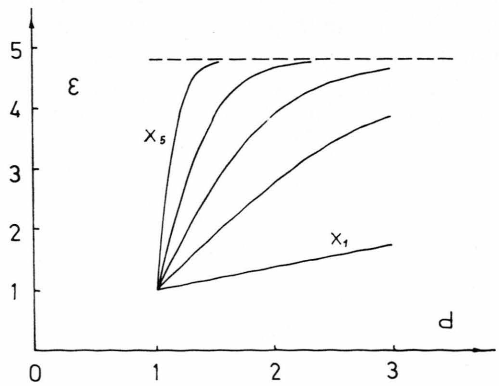

Fig. 5. The effective permittivity as a function of the wall thickness parameter; x takes the values 0.1 (label x_1), 0.5, 1, 3, and 5 (label x_5); the dashed line: $\varepsilon_g = 4.8$.

Taking into account the known formula arctanh $(z) = 0.5 \ln[(1+z)/(1-z)]$, we get at once

$$d_+ = 1 + \frac{1}{2x} \ln\left(19 - \frac{20}{\varepsilon_g}\right). \tag{7}$$

Accepting $\varepsilon_g = 4.8$, we obtain $d_+ \simeq 1 + 1.35/x$. Now we see that any case with $d > d_+$ is in fact a surface wave thick-wall guiding structure. The rule is practical and simple: The tube of a given parameter d supporting the surface wave propagation can be treated as a homogeneous dielectric of the permittivity $\varepsilon_g = 4.8$ if the condition $d > (1 + 1/x)$ is satisfied for the given wavelength $\lambda = 2\pi a/x$.

The thin-wall approximation. Let us now put $\varepsilon_- = 1.1$. In order to satisfy the equality $\varepsilon = \varepsilon_-$ we must choose the parameter d_- to be

$$d_- = 1 + \frac{1}{2x} \ln\left(\frac{\varepsilon_g - 0.9}{\varepsilon_g - 1.1}\right). \tag{8}$$

So, any tube with $d < d_-$ can be classified as a thin-wall guiding structure. With $\varepsilon_g = 4.8$ the equation reduces to $d_- \simeq [1 + 1/40x)]$, and again the rule is simple: the tube supporting the surface wave propagation can be totally ignored if $d_- < [1 + 1/(40x)]$; essentially, we have the propagation of a surface wave along a bare plasma column in vacuum.

5. Conclusion

The azimuthally symmetric surface wave propagating on a plasma column placed in a glass tube of finite thickness may be conceived as a wave propagating on a plasma column placed in an equivalent homogeneous dielectric of given effective permittivity (s. (4)). This statement is true in the sense that the dispersion relations of the two waves should be the same. In the case of the quasistatic approximation the agreement is complete. Using the full dispersion relations, the method gives correct semi-quantitative results. The concept of the equivalent dielectric enables a considerable reduction of involved mathematical expressions, whenever the direct use of the dispersion relations is necessary [8, 9]. Also, in that way we may quickly obtain some qualitative answers concerning the behaviour of the bulky full dispersion relation explained in the Appendix. The proposed method should be effective, too, when applied on the dipole mode of the surface wave. Preliminary computations show the expected agreement in a large-x region. We believe that for small-x values our approximation could even improve some of defects known to exist when the quasistatic approximation is in question. The work is in progress and will be presented elsewhere.

Appendix

The Dispersion of an Axially Symmetric Electromagnetic Surface Wave

Consider the surface wave propagation along an infinitely long cylindrical plasma column of radius a. The plasma is uniform and isotropic. In the cold plasma approximation the permittivity is $\varepsilon_p = 1 - \omega^2/\omega_p^2$, where ω is the angular frequency of the wave and ω_p the angular plasma frequency. Denote with b the outer glass tube radius ($\delta = b - a$ is the tube thickness). The permittivity of glass is ε_g. This guiding structure is placed in air. Now, we must look for the surface wave solution applying the full set of Maxwell's equations. In the linear theory, all the field components are of the form

$$g = g_0 + g_1, \tag{A1}$$

where g_0 is the equilibrium part. The perturbation part $g_1 \ll g_0$ can be represented in cylindrical coordinates as

$$g_1(r, \theta, z, t) = g_1(r)\, e^{i(\omega t - \beta z - n\theta)}. \tag{A2}$$

Here β is the axial and n the azimuthal wavenumber. The Maxwell functions yield g_1(r), the proper radial part of all the needed field components. In fact, in our symmetrical case with $n = 0$ it is convenient to solve at the beginning the wave equation for the potential

$$\frac{\partial^2 \phi_i}{\partial \varrho^2} + \frac{1}{\varrho}\frac{\partial \phi_i}{\partial \varrho} + \frac{\partial^2 \phi_i}{\partial z^2} + k_0^2 \varepsilon_i = 0 \tag{A3}$$

in the three regions, and than apply the relations

$$E_\varrho = \frac{i}{\omega \varepsilon} \frac{\partial \phi}{\partial z}, \quad H_\varphi = \phi,$$

$$E_z = -\frac{i}{\omega e}\left[\frac{\partial \phi}{\partial \varrho} + \frac{\phi}{\varrho}\right]. \tag{A4}$$

Of course, it is necessary to match the wave solutions at the interfaces plasma/glass and glass/air. This procedure means that one must apply the electromagnetic boundary conditions (the choice may be $(H_\varphi)_+ = (H_\varphi)_+$ and $(\varepsilon E_\varrho)_- = (\varepsilon E_\varrho)_+$ at $r = a$ and $r = b$;

this set of equations suffices to eliminate the four integration constants). As a result, one gets the dispersion relation, the equation which relates ω with λ (the surface wave wavelength) or β (the wavenumber $\beta = 2\pi/\lambda$).

In order to write down the dispersion relation we must first define the next set of abbrevations:

$$A_1 = u_1 a, \quad A_2 = u_2 a, \quad B_2 = u_2 b, \quad B_3 = u_3 b, \tag{A5}$$

where $u_i = [\beta^2 - (\omega^2/c_0^2)\,\varepsilon_i]^{0.5}$. The index i takes the values 1, 2, and 3. The permittivities are $\varepsilon_1 = \varepsilon_p$, $\varepsilon_2 = \varepsilon_g$, $\varepsilon_3 = 1$. Here, c_0 is the speed of light in vacuum. Also, let us construct the quadratic 4×4 matrix $M = (m_{ij})_1^4$, having the next nonzero coefficients:

$$\begin{aligned} &m_{11} = I_1(A_1),\ m_{12} = -\mathrm{U}_1(A_2),\ m_{13} = -K_1(A_2),\\ &m_{22} = I_1(B_2),\ m_{23} = K_1(B_2),\ m_{24} = -K_1(B_3),\\ &m_{31} = (A_1/\varepsilon_1)\,I_0(A_1),\ m_{32} = (-A_2/\varepsilon_2)\,I_0(A_2),\\ &m_{33} = (A_2/\varepsilon_2)\,K_0(A_2),\ m_{42} = (B_2/\varepsilon_2)\,I_0(B_2),\\ &m_{43} = (-B_2/\varepsilon_2)\,K_0(B_2),\ m_{44} = (B_3/\varepsilon_3)\,K_0(B_3). \end{aligned} \tag{A6}$$

Here, I and K are the modified Bessel functions of the first and second kind, of indicated orders and arguments. Denoting $\Delta_0(\omega, \beta) = \det(M)$, we can obtain the searched dispersion relation equating the determinant with zero,

$$\Delta_0(\omega, \beta) = 0. \tag{A7}$$

In an expanded form, the above expression is quoted in (1).

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